

A Lagrangian for self-dual strings

Vasilis Niarchos

*Crete Center for Theoretical Physics and
Crete Center for Quantum Complexity and Nanotechnology
Department of Physics, University of Crete, 71303, Greece*

niarchos@physics.uoc.gr

Abstract

We propose a Lagrangian for the low-energy theory that resides at the $(1+1)$ -dimensional intersection of N semi-infinite M2-branes ending orthogonally on M M5-branes in $\mathbb{R}^{1,2} \times \mathbb{C}^4/\mathbb{Z}_k$ (for arbitrary positive integers N, M, k). We formulate this theory as a 2d boundary theory with explicit $\mathcal{N} = (1, 1)$ supersymmetry that contains two superfields in the bi-fundamental representation of $U(N) \times U(M)$ interacting with the $(2+1)$ -dimensional $U(N)_k \times U(N)_{-k}$ ABJM Chern-Simons-matter theory in the bulk. We postulate that the boundary theory exhibits in the deep infrared supersymmetry enhancement to $\mathcal{N} = (4, 2)$, or $\mathcal{N} = (4, 4)$ depending on the value of k . Arguments in favor of the proposal follow from the study of the open string theory of a U-dual type IIB Hanany-Witten setup. To formulate the bulk-boundary interactions special care is taken to incorporate all the expected boundary effects on gauge symmetry, supersymmetry, and other global symmetries.

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1 Introduction

Since M2-branes can end on M5-branes it has long been suspected that the M5-brane theory is a still illusive six-dimensional non-critical string theory. The strings of this theory are charged under a self-dual three-form field strength, hence they are frequently referred to as self-dual strings. When the M5-branes are coincident the theory is non-abelian and intrinsically strongly coupled. As a result, it has proven a very hard problem to formulate this string theory and to extract directly information about the quantum physics of M5-branes.

Clearly, the two-dimensional intersection of M2-branes ending on M5-branes is at the heart of this problem. It would be useful to understand precisely the degrees of freedom that reside on this intersection and how they interact with the three-dimensional and six-dimensional bulk on the M2 and M5-branes respectively. It is sensible to analyse this problem first in a symmetric configuration, e.g. the half-BPS configuration of N coincident M2-branes (extended along the half-plane $x^2 > 0$) that end orthogonally on M coincident M5-branes

$$\begin{array}{ll}
 N \text{ M2} & : \quad 0 \quad 1 \quad 2_+ \\
 M \text{ M5} & : \quad 0 \quad 1 \quad \quad \quad 7 \quad 8 \quad 9 \quad 10
 \end{array} \tag{1}$$

In flat space the two-dimensional theory at the intersection enjoys large $\mathcal{N} = (4, 4)$ supersymmetry. To date there has been very limited information about this theory. Let us summarize quickly some of the most prominent developments that are pertinent for this paper.

From the M5-brane point of view the orthogonal M2-branes can be viewed as a string soliton spike. The first successful description of this soliton (as an M-theory BIon) was given by Howe, Lambert and West [1] for a single M5-brane. A similar analysis for coincident M2 and M5-branes in the large- N, M limit was performed in [2] using a holographic supergravity analysis based on the blackfold approach [3, 4]. A noteworthy result of that work was a specific prediction for the leading behavior of the central charge c of the putative two-dimensional superconformal theory at the intersection [5, 6]

$$c \sim \frac{N^{\frac{3}{2}}}{\sqrt{\lambda}} + \dots \sim \frac{M^3}{\lambda^2} + \dots \quad (2)$$

in a 't Hooft-like limit where $N, M \gg 1$ with the ratio $\lambda = \frac{M^2}{N}$ fixed and large. The dots indicate subleading terms in a $1/\lambda$ expansion. The appearance of the powers $N^{\frac{3}{2}}$ and M^3 in the two expressions on the rhs of (2) is suggestive of a close relation to the well-known scaling of massless degrees of freedom of M2-branes ($N^{\frac{3}{2}}$), and M5-branes (M^3). Different expressions for c in other regimes were derived in [7] using anomaly considerations in the Coulomb branch of the M5-brane theory.¹

Let us note in passing that an exact fully localized half-BPS supergravity solution that describes the M2-M5 configuration (1) in flat space is currently not known. For an older attempt to this problem we refer the reader to [9]. A more recent analysis of AdS solutions that are presumably near-horizon limits of M2-M5 configurations was performed in [10]. It would be interesting to distill further information about the quantum properties of the M2-M5 intersections from such asymptotically AdS solutions in supergravity.

There have also been several attempts to analyze the field theory of the intersection (1) from the viewpoint of the M2-branes. An M-theory generalization of the Nahm equations for the BIon was proposed by Basu and Harvey in [11]. Subsequently, with the advent of the ABJM model [12], the low-energy theory on N M2-branes was formulated as a $U(N) \times U(N)$ Chern-Simons-matter theory with explicit $\mathcal{N} = 6$ supersymmetry. The properties of semi-infinite M2-branes ending on M M5-branes are captured from this perspective by appropriate boundary conditions and/or appropriate boundary degrees of freedom in the ABJM model on a half-plane.

The effects of boundaries in supersymmetric Chern-Simons-matter theories were considered by several authors. A formulation of the boundary problem in the context of the M2-M5 system in terms of supersymmetric boundary conditions was put forward in [13]. Other authors considered an alternative formulation that employs suitable boundary degrees of freedom. With emphasis on the boundary effects on supersymmetry Ref. [14] considered possible boundary in-

¹For an interesting observation on the role of self-dual string junctions in the Coulomb phase of the ADE 6d (2,0) superconformal fields theories and the problem of the M^3 scaling of the massless degrees of freedom on the M5-branes see [8].

interactions in $\mathcal{N} = 2$ Chern-Simons-matter theories using the technology of [15]. A different set of boundary interactions, that emphasized the role of gauge symmetry, was considered in [16, 17]. Although both approaches in this direction are technically relevant for the M2-M5 system, their precise implementation to this problem has been obscure, because a clear M-theory guide to the boundary degrees of freedom and interactions that are needed to describe the M2-M5 system was mostly lacking. A specific proposal towards the resolution of these issues will be the main contribution of this paper.

Finally, in more recent developments it has proven useful to consider configurations of intersecting M2 and M5-branes with compactified worldvolumes. In this context a computation of the elliptic genus of M2-branes suspended between parallel M5-branes was performed in [18, 19, 20], and [21]. Ref. [22] considered M5-branes wrapped around punctured Riemann surfaces. In this setup the M2-branes realize surface operators in four-dimensional $\mathcal{N} = 2$ field theories.

Main contribution and brief summary of the paper

The approach we take in this paper is particularly simple. The successful formulation of the low-energy theory on multiple M2-branes as supersymmetric Chern-Simons-matter theory, [12], relied on a U-dual description of M2-branes as D3-branes suspended between appropriate stacks of 5-branes in a type IIB Hanany-Witten setup. In section 2 we describe how to incorporate an extra stack of M D5-branes in this setup, where N D3-branes can end on a half-BPS $(1+1)$ -dimensional boundary. We show that the new configuration lifts in M-theory to the M2-M5 system of interest probing a $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity. For $M = 0$ D5-branes our setup reduces to the well-known brane configuration of [12].

In section 3 we use the type IIB setup to read off the spectrum and interactions at the D3-D5 boundary. We find that the massless boundary degrees of freedom that arise in the D3-D5 open string theory are two sets of 2d $\mathcal{N} = (1, 1)$ supermultiplets in the bi-fundamental representation of the $U(M) \times U(N)$ group. Using a formulation with explicit $\mathcal{N} = 2$ supersymmetry in the three-dimensional bulk we present a 2d boundary theory that exhibits $\mathcal{N} = (1, 1)$ supersymmetry. Precise bulk-boundary interactions of this theory are proposed using the recent results of Ref. [23], that is building on the previous works [15, 16, 17]. Analyzing the symmetries of the postulated action and the symmetries of the underlying brane setup we postulate that for generic Chern-Simons level $k > 2$ the bulk-boundary theory flows in the deep infrared to a fixed point with the expected 2d $\mathcal{N} = (4, 2)$ supersymmetry. We anticipate a further enhancement of the boundary supersymmetry for the special value $k = 1$ to large $\mathcal{N} = (4, 4)$. A similar enhancement for $k = 2$ is possible but even less obvious at the moment (see comments in section 2).

We conclude in section 4 with a brief discussion of interesting aspects of the proposed action

and its implications in M-theory. Open problems that are worth pursuing further are also discussed in this section.

2 M2-M5 from the M-theory lift of a type IIB setup

2.1 Type IIB setup

Our starting point is the following Hanany-Witten setup in type IIB string theory that realizes at low energies the ABJM model [12] on a space with a boundary

$$\begin{array}{rcll}
N \text{ D3} & : & 0 & 1 & 2_+ & & & & 6_+ \\
N \text{ D3}' & : & 0 & 1 & 2_+ & & & & 6_- \\
1 \text{ NS5} & : & 0 & 1 & 2 & 3 & 4 & 5 & \\
1 \text{ (1, } k \text{)5} & : & 0 & 1 & 2 & \begin{bmatrix} 3 \\ 7 \end{bmatrix}_\theta & \begin{bmatrix} 4 \\ 8 \end{bmatrix}_\theta & \begin{bmatrix} 5 \\ 9 \end{bmatrix}_\theta & \\
M \text{ D5} & : & 0 & 1 & & & & & 6 & 7 & 8 & 9
\end{array} \tag{3}$$

In this setup an NS5-brane and a $(1, k)$ 5-brane bound state² are located at antipodal points on the S^1 direction $x^6 \in [-\pi, \pi)$. The angle θ is fixed by supersymmetry in terms of the complex axion-dilaton coupling τ

$$\theta = \arg(\tau) - \arg(k + \tau), \quad \tau = \frac{i}{g_s} + \chi \tag{4}$$

where g_s is the string coupling constant and χ the value of the axion field (that we set to zero). Two stacks of D3-branes are suspended between the NS5 and $(1, k)$ 5-branes along the directions (0126): N D3 branes wrap the semi-circle $x^6 \in (0, \pi)$, and N D3' branes wrap the semi-circle $x^6 \in (-\pi, 0)$. The setup of D3-NS5- $(1, k)$ 5 branes, with the D3-branes stretching infinitely across the whole (012) plane and $M = 0$ D5-branes, is the original configuration of Ref. [12] that formulated the low-energy theory on N M2-branes probing $\mathbb{C}^4/\mathbb{Z}_k$ as a $U(N)_k \times U(N)_{-k}$ Chern-Simons-matter theory. For $k > 2$ this theory is an $\mathcal{N} = 6$ three-dimensional gauge theory. For $k = 1, 2$ there is an infrared enhancement of supersymmetry to $\mathcal{N} = 8$ [12, 24].

Compared to Ref. [12], the setup (3) introduces an additional stack of M D5-branes (last line in (3)) that intersect the N pairs of D3-branes on a two-dimensional boundary along the plane (01). The semi-infinite D3-branes stretch on the half-line $x^2 > 0$ and end on the D5-branes at $x^2 = 0$ (hence the notation 2_+ in the list of the configuration (3)). From the low-energy point of view, the D5-branes introduce a boundary on the three-dimensional Chern-Simons-matter

²We will be using conventions where (p, q) 5 refers to a fivebrane bound state with p units of NS5-brane charge and q units of D5-brane charge. Moreover, without loss of generality we will henceforth assume that $k > 0$. The notation $\begin{bmatrix} a \\ b \end{bmatrix}_\theta$ denotes that a brane is oriented along the direction $\cos \theta x^a + \sin \theta x^b$ in the (x^a, x^b) plane.

theory that resides on the D3-branes. One can verify by explicit computation (see e.g. appendix A of Ref. [23] for a related discussion) that the brane setup (3) preserves 3 real supersymmetries —2 left-moving and 1 right-moving. Hence this is a non-chiral half-BPS boundary. At low-energies the global symmetries of the M-theory lift (to be discussed momentarily) suggest the infrared enhancement of supersymmetry to $\mathcal{N} = (4, 2)$ in two dimensions. In the special case where $k = 1$ they suggest a further enhancement to large $\mathcal{N} = (4, 4)$.

As an aside remark, it is useful to note here, for later purposes, the following fact. Rotating the $(1, k)5$ -brane in (3) along the more general orientation $\left(\begin{bmatrix} 3 \\ 7 \end{bmatrix}_\psi \quad \begin{bmatrix} 4 \\ 8 \end{bmatrix}_\psi \quad \begin{bmatrix} 5 \\ 9 \end{bmatrix}_\theta \right)$ further reduces the explicit supersymmetry from 3 real supersymmetries to 2 real supersymmetries when $\psi \neq \theta$. Namely, changing ψ reduces $\mathcal{N} = (2, 1) \rightarrow \mathcal{N} = (1, 1)$ in two dimensions.

In section 3 we consider the low-energy field theory at the D3-D5 intersection following a recent similar discussion of open string dynamics in [23]. In the rest of this section we elaborate further on the M-theory lift of the setup (3) and its relation to the orthogonal M2-M5 intersection which is the system of main interest in this paper.

2.2 M-theory lift

Repeating the steps of the U-duality transformation in [12] we first perform a T-duality transformation along the direction 6. This results to a type IIA brane configuration on a space with a T-dual S^1 direction $\tilde{6}$:

$$\begin{array}{llllllll}
N \ D2_+ & : & 0 & 1 & 2_+ & & & \\
1 \ KK_{\tilde{6}} & : & 0 & 1 & 2 & 3 & 4 & 5 \\
1 \ (KK_{\tilde{6}} - k \ D6) & : & 0 & 1 & 2 & \begin{bmatrix} 3 \\ 7 \end{bmatrix}_\theta & \begin{bmatrix} 4 \\ 8 \end{bmatrix}_\theta & \begin{bmatrix} 5 \\ 9 \end{bmatrix}_\theta \\
M \ D4 & : & 0 & 1 & & & 7 & 8 & 9
\end{array} \tag{5}$$

The notation $KK_{\tilde{6}}$ refers to a Kaluza-Klein (KK) monopole associated with the dual circle $\tilde{6}$. Similarly $(KK_{\tilde{6}} - k \ D6)$ refers to a KK monopole with k units of D6-brane flux.

Next we lift to M-theory by adding the 11-th (M-theory) direction x^{10} . The N D2-branes ending on M D4-branes become N M2-branes ending on M M5-branes. The KK monopole $KK_{\tilde{6}}$ remains a KK monopole associated with $\tilde{6}$ and the $(KK_{\tilde{6}} - k \ D6)$ bound state becomes a KK monopole associated with a linear combination of the circles $\tilde{6}$ and 10. At the intersection of the two KK monopoles the eight-dimensional space transverse to the plane (012) becomes the orbifold $\mathbb{C}^4/\mathbb{Z}_k$ [12].

To summarize, after the above U-duality transformation we obtain the orthogonal M2-M5 intersection

$$\begin{array}{llllll}
N \ M2_+ & : & 0 & 1 & 2_+ & \\
M \ M5 & : & 0 & 1 & & 7 & 8 & 9 & 10
\end{array} \tag{6}$$

probing the $\mathbb{C}^4/\mathbb{Z}_k$ singularity in the (3456789(10)) directions. The $k = 1$ case reduces to the familiar M2-M5 intersection in flat space.

As explained in appendix B of Ref. [12] the metric of the transverse eight-dimensional space takes the form of a toric hyperkähler manifold with a diagonal 2×2 matrix of U -functions in the coordinates

$$\vec{x}'_1 = (x^7, x^8, x^9), \quad \varphi'_1 = x^{\tilde{6}} - \frac{1}{k}x^{10} \quad (7)$$

and

$$\vec{x}'_2 = (x^7 + kx^3, x^8 + kx^4, x^9 + kx^5), \quad \varphi'_2 = \frac{1}{k}x^{10}. \quad (8)$$

The coordinates (φ'_1, φ'_2) have periodicity 2π plus the orbifold identification

$$(\varphi'_1, \varphi'_2) \sim (\varphi'_1, \varphi'_2) + \left(-\frac{1}{k}, \frac{1}{k}\right). \quad (9)$$

In the absence of the M5-branes the overall symmetry of the transverse space is $SO(6) \times SO(2)$. $SO(6)$ is associated with transformations in the (345789) directions and $SO(2)$ with translations of x^{10} , i.e. with translations $(\varphi'_1, \varphi'_2) \rightarrow (\varphi'_1 - \varphi, \varphi'_2 + \varphi)$.

In the presence of the M5-branes the $SO(6)$ in the (345789) directions breaks to $SO(3) \times SO(3)$ transformations that are either fully parallel to the M5-brane worldvolume or fully orthogonal. Since $SO(3) \simeq SU(2)$ and $SU(2) \times SU(2) \simeq SO(4)$, the total symmetry of the M2-M5 configuration in the presence of the orbifold, for $k > 2$, is $SO(4) \times SO(2)$. This is an R-symmetry for the two-dimensional theory at the M2-M5 intersection. Its presence suggests that the infrared theory at the intersection exhibits $\mathcal{N} = (4, 2)$ supersymmetry.

For $k = 1$ the symmetry of the transverse \mathbb{R}^8 is $SO(4) \times SO(4)$ from the separate rotation symmetries of the two orthogonal \mathbb{R}^4 's in \mathbb{R}^8 . This symmetry is the expected R-symmetry group of a 2d CFT with large $\mathcal{N} = (4, 4)$ superconformal algebra.

The case with $k = 2$ is potentially more interesting. In the absence of the M5-branes arguments were given in [12] for the quantum mechanical enhancement of the R-symmetry group in the three-dimensional Chern-Simons-matter theory from $SO(6)$ to $SO(8)$. In our setup a stack of M5-branes intersects the \mathbb{Z}_2 singularity. If the non-abelian interactions of the M5-brane theory exhibit the same global symmetry enhancement one would expect an $SO(4) \times SO(4)$ symmetry for the M2-M5 intersection also at $k = 2$. It is currently unclear to us if this enhancement actually takes place.

3 ABJM on a space with boundary from open string theory

In this section we focus on the open string theory dynamics of the type IIB setup (3). Following the discussion of the recent paper [23] we propose a specific action for the 3d-2d bulk-boundary

dynamics at the D3-D5 intersection.

3.1 3d bulk

The 3d bulk theory, which arises as the IR effective field theory description of the open string dynamics on the D3-branes in the setup (3), is the $\mathcal{N} = 6$ $U(N)_k \times U(N)_{-k}$ ABJM theory. It is formulated most conveniently as an $\mathcal{N} = 2$ theory with appropriate matter representations. The Lagrangian for the $\mathcal{N} = 2$ vector multiplet consists of the $\mathcal{N} = 2$ Chern-Simons (CS) theory at level k , and the $\mathcal{N} = 2$ CS theory at level $-k$. The gauge group of both CS theories is $U(N)$. To distinguish between the two gauge groups we will denote them as $U(N)_+$ (with CS level $+k$), and $U(N)_-$ (with CS level $-k$).

The matter content of the theory consists of 2 chiral superfields A^a ($a = 1, 2$) in the bi-fundamental representation of $U(N)_+ \times U(N)_-$ and 2 chiral superfields B_a ($a = 1, 2$) in the anti-bifundamental representation. (The complex conjugate anti-chiral superfields will be denoted with a bar.) It is convenient, and most appropriate from the point of view of the brane configuration (3), to include two massive $\mathcal{N} = 2$ chiral superfields ϕ_{\pm} with superpotential

$$W = \frac{k}{8\pi} \text{Tr} [\phi_+^2 - \phi_-^2] + \text{Tr} [B_a \phi_+ A^a] + \text{Tr} [A^a \phi_- B_a] . \quad (10)$$

Integrating out the massive superfields sets

$$\phi_+ = -\frac{4\pi}{k} A^a B_a , \quad \phi_- = \frac{4\pi}{k} B_a A^a \quad (11)$$

and leads in the deep IR to the quartic superpotential

$$W = \frac{4\pi}{k} \text{Tr} [A^1 B_1 A^2 B_2 - A^1 B_2 A^2 B_1] = \frac{2\pi}{k} \varepsilon_{ab} \varepsilon^{\dot{a}\dot{b}} \text{Tr} [A^a B_{\dot{a}} A^{\dot{b}} B_b] \quad (12)$$

which is responsible for the supersymmetry enhancement to $\mathcal{N} = 6$. ε_{ab} is the totally anti-symmetric symbol; we use $\varepsilon_{12} = 1$.

It is useful to highlight the following points regarding the $\mathcal{N} = 2$ formulation of the ABJM theory:

- (i) A general mass $m \neq \frac{k}{8\pi}$ in (10) corresponds in the brane setup (3) to a general angle $\psi \neq \theta$ for the $(1, k)$ 5-brane oriented along $\left(\begin{bmatrix} 3 \\ 7 \end{bmatrix}_{\psi} \quad \begin{bmatrix} 4 \\ 8 \end{bmatrix}_{\psi} \quad \begin{bmatrix} 5 \\ 9 \end{bmatrix}_{\theta} \right)$, [25, 26]. As we pointed out near the end of subsection 2.1, and is visible from (10), for $\psi \neq \theta$ and $m \neq \frac{k}{8\pi}$ the explicit supersymmetry of the 3d bulk theory is $\mathcal{N} = 2$ (and therefore 2d $\mathcal{N} = (1, 1)$ on a half-BPS boundary). Nevertheless, it was shown perturbatively in [27] in the large k limit that the $\mathcal{N} = 6$ fixed point is an attractor of the RG flow in the 3d Chern-Simons-matter theory, so different values of m in the bare action do not affect the IR physics crucially in the

bulk. It is natural to expect a similar effect for all values of k . For technical reasons that will become clear momentarily, it will be useful to work with a general mass m in the bulk superpotential

$$W = m \text{Tr} [\phi_+^2 - \phi_-^2] + \text{Tr} [B_a \phi_+ A^a] + \text{Tr} [A^a \phi_- B_a] . \quad (13)$$

- (ii) The bare $\mathcal{N} = 2$ supersymmetric action with superpotential (10) does not exhibit the $\mathcal{N} = 3$ supersymmetry automatically in the non-abelian case unless some of the auxiliary fields in the $\mathcal{N} = 2$ supersymmetric multiplets are integrated out. Hence, we would not expect to see the full $\mathcal{N} = (2, 1)$ supersymmetry on the half-BPS 2d boundary in the UV in the above language in a fully $\mathcal{N} = 2$ super-gauge invariant formulation. Note that the abelian case does not exhibit this issue.
- (iii) Finally, working with explicit $\mathcal{N} = 2$ supersymmetry in the bulk allows us to circumvent an important technical issue that has to do with the effects of the boundary. It is well-known that boundaries break the super-gauge invariance of supersymmetric gauge theories. Therefore, the passage to a preferable gauge may be inappropriate in the presence of a boundary. As a result, a proper treatment of boundaries typically requires a formulation with full off-shell supersymmetry. For example, in the case of the $\mathcal{N} = 6$ Chern-Simons-matter theories of interest this would require the use of an explicit $\mathcal{N} = 6$ formalism, which is a rather complicated task.

We circumvent this problem by formulating the half-BPS boundary and the corresponding bulk-boundary interactions in the bare $\mathcal{N} = 2$ Lagrangian with superpotential (13). Then by tuning the bare mass m to the $\mathcal{N} = 3$ point $m = \frac{k}{8\pi}$, or by just allowing the renormalization group to flow to the $\mathcal{N} = 6$ fixed point in the bulk, we postulate that our half-BPS boundary flows accordingly from a UV $\mathcal{N} = (1, 1)$ point to the desired IR point with $\mathcal{N} = (4, 2)$ (or $\mathcal{N} = (4, 4)$) supersymmetry. We provide favorable evidence for this conjecture using the available information from string theory and by checking explicitly that the postulated action has the expected global symmetries.

3d bulk action

In the brane setup (3) there is a boundary for the 3d theory at $x^2 = 0$. Accordingly, we will formulate the ABJM theory on the half-plane at $x^2 > 0$. We use the $\mathcal{N} = 2$ superspace formalism and the set of conventions summarized in [23].³ In these conventions the content of

³The superspace coordinates are $(x^\mu, \vartheta_\alpha)$ with spacetime indices $\mu = 0, 1, 2$ and spinor indices $\alpha = \pm$. The odd Grassmann variables ϑ_α are complex: $\vartheta_\alpha = \frac{1}{\sqrt{2}} (\theta_{1\alpha} + i\theta_{2\alpha})$. $\theta_{s\alpha}$ ($s = 1, 2$) are real Grassmann odd variables in $\mathcal{N} = 1$ superspace. We follow the $\mathcal{N} = 1$ superspace conventions in Ref. [28].

the $\mathcal{N} = 2$ vector multiplet is summarized in an $\mathcal{N} = 2$ vector superfield V that contains the three-dimensional gauge field A_μ , several auxiliary scalars and their supersymmetric partners. As we mentioned already, in ABJM there are two vector multiplets that we call V_+ , V_- , which appear in the CS actions with level $+k$ and $-k$ respectively. The fully covariant formulation of the $\mathcal{N} = 2$ CS theory is four-dimensional [29]. In our context

$$\begin{aligned} \mathcal{S}_{CS}[V_+, V_-] &= -\frac{k}{2\pi} \int_0^1 ds \int d^3x \int d^4\vartheta \operatorname{Tr} [V_+ \bar{D}^\alpha (e^{sV_+} D_\alpha e^{-sV_+})] \\ &+ \frac{k}{2\pi} \int_0^1 ds \int d^3x \int d^4\vartheta \operatorname{Tr} [V_- \bar{D}^\alpha (e^{sV_-} D_\alpha e^{-sV_-})] . \end{aligned} \quad (14)$$

D_α is the $\mathcal{N} = 2$ superspace covariant derivative

$$D_\alpha = \partial_\alpha + (\gamma^\mu \bar{\vartheta})_\alpha \partial_\mu , \quad \bar{D}_\alpha = \bar{\partial}_\alpha + (\gamma^\mu \vartheta)_\alpha \partial_\mu . \quad (15)$$

The matter sector interactions include the superpotential interactions (13) (or (10) for the specific orientations in (3))

$$\mathcal{S}_W[\phi_\pm, A, B] = \int d^3x d^2\vartheta W + \text{c.c.} , \quad (16)$$

and the Kähler interactions that provide the kinetic terms. For simplicity, we will consider here canonical Kähler interactions. Note however, that, unlike the superpotential interactions, the Kähler interactions receive quantum corrections. Accordingly, the boundary interactions that will be formulated shortly have to be adjusted suitably to take into account these quantum corrections in order to preserve the desired amount of supersymmetry. This can be performed straightforwardly with the prescription that will be described in a moment. The canonical Kähler interactions that we consider here are

$$\mathcal{S}_K[\phi_\pm, A, B, V_\pm] = \int d^3x \int d^4\vartheta \operatorname{Tr} [\bar{\phi}_+ e^{V_+} \phi_+ + \bar{\phi}_- e^{V_-} \phi_- + \bar{A}_a e^{V_+} A^a e^{-V_-} + \bar{B}^a e^{V_-} B_a e^{-V_+}] . \quad (17)$$

In summary, the total bulk action is

$$\mathcal{S}_{bulk} = \mathcal{S}_{CS}[V_+, V_-] + \mathcal{S}_K[\phi_\pm, A, B, V_\pm] + \mathcal{S}_W[\phi_\pm, A, B] . \quad (18)$$

3.2 2d boundary

The boundary theory of a Chern-Simons-matter theory is not unique. In previous explorations of the subject [14, 16, 17] boundary interactions were formulated with two main guiding criteria: the restoration of the appropriate amount of gauge invariance and supersymmetry. Even with these criteria there is still considerable freedom on the choice of boundary degrees of freedom

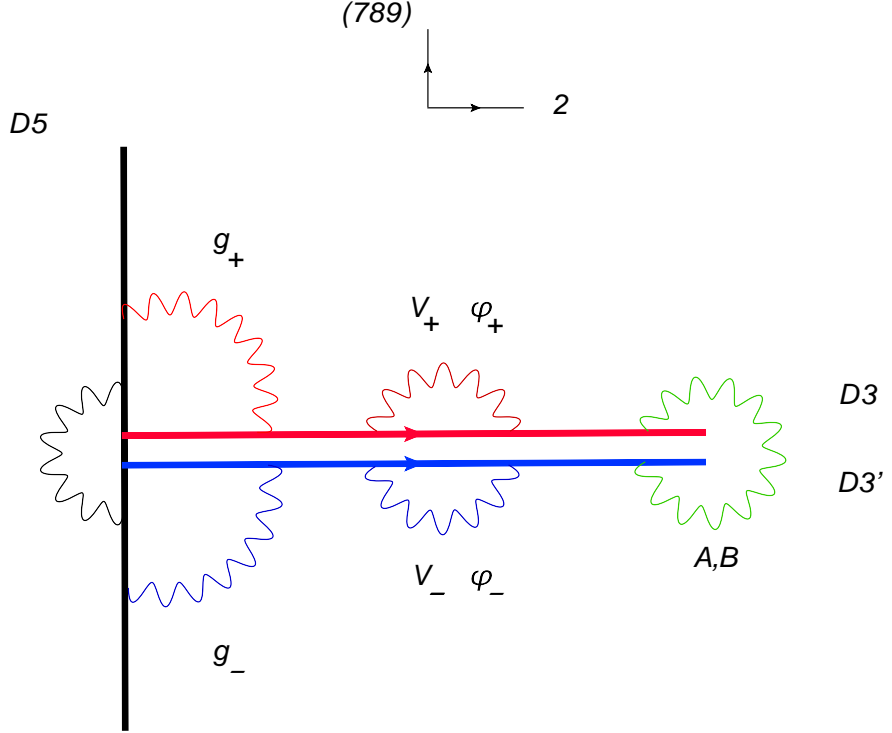


Figure 1: Two stacks of N D3-branes (wrapping different halves of the 6-circle) end from the right ($x^2 > 0$) on M D5-branes within the brane configuration (3). We have isolated the D3-D5 part of the intersection in the directions (2789) leaving the NS5, $(1, k)5$ part of the setup implicit.

and boundary/bulk-boundary interactions. As a result, a well-motivated specific proposal for the boundary theory of N M2-branes ending on M M5-branes has not been possible so far.

In the context of the brane setup (3) we find ourselves in a much better situation. Following the recent discussion in [23] we can now use the open string theory of the type IIB Hanany-Witten setup as a concrete guide towards a boundary action. Different sectors of the open string theory at the D3-D5 intersection are summarized Fig. 1. Besides the 3d bulk fields V_{\pm} , ϕ_{\pm} , A^a , B_a from 3-3, 3'-3', or 3-3' open strings there are also g_+ fields from 3-5 (red) strings and g_- fields from 5-3' (blue) strings. Both are 2d $\mathcal{N} = (1, 1)$ superfields; g_+ is in the bifundamental representation of $U(N)_+ \times U(M)$ and g_- in the bifundamental of $U(M) \times U(N)_-$. There are also fields from 5-5 strings (black color) which will be ignored since their dynamics is irrelevant at low energies. The group representations in which different supermultiplets belong are summarized in Table 1.

The first step in the construction of a boundary action involves the addition of suitable

Superfield	$U(N)_-$	$U(N)_+$	$U(M)$
V_+	1	adjoint	1
V_-	adjoint	1	1
ϕ_+	1	adjoint	1
ϕ_-	adjoint	1	1
A^a	\square	$\overline{\square}$	1
B_a	$\overline{\square}$	\square	1
g_+	1	$\overline{\square}$	\square
g_-	\square	1	$\overline{\square}$

Table 1: A summary of group representations.

boundary interactions that restore the desired amount of supersymmetry. In the case at hand we have explicit $\mathcal{N} = 2$ supersymmetry in the bulk and want to restore $\mathcal{N} = (1, 1)$ supersymmetry on the boundary. Applying the prescription of Ref. [15] to a general $\mathcal{N} = 2$ action (expressed conveniently in $\mathcal{N} = 1$ superspace language)

$$\mathcal{S} = \int d^3x d^2\theta_1 d^2\theta_2 \mathcal{L}(x^\mu, \theta_1, \theta_2) \quad (19)$$

we obtain the action

$$\mathcal{S}^{(1,1)} = \int d^3x \left\{ d^2\theta_1 d^2\theta_2 \mathcal{L} - d^2\theta_1 \partial_2 \mathcal{L} \Big|_{\theta_2=0} + d^2\theta_2 \partial_2 \mathcal{L} \Big|_{\theta_1=0} - \partial_2 \partial_2 \mathcal{L} \Big|_{\theta_1=\theta_2=0} \right\} \quad (20)$$

that preserves the supersymmetries generated by (Q_{1+}, Q_{2-}) . In our case, $\mathcal{S} = \mathcal{S}_{bulk}$ (see eq. (18)). We will denote this boundary-corrected version of \mathcal{S}_{bulk}

$$\mathcal{S}_{bulk}^{(1,1)} = \mathcal{S}_{CS}^{(1,1)} + \mathcal{S}_K^{(1,1)} + \mathcal{S}_W^{(1,1)}. \quad (21)$$

The next step involves the incorporation of the boundary multiplets g_\pm in a manner that restores the broken $U(N) \times U(N)$ gauge invariance at the boundary. Following [23] we extend the definition of g_\pm in the bulk as 3d $\mathcal{N} = 2$ superfields (denote them \mathbf{g}_\pm), and define the $U(M)$ $\mathcal{N} = 2$ vector superfields $\mathbf{V}_\pm^{g_\pm}$

$$e^{\mathbf{V}_+^{g_+}} = \tilde{\mathbf{g}}_+ e^{V_+} \mathbf{g}_+, \quad e^{\mathbf{V}_-^{g_-}} = \tilde{\mathbf{g}}_- e^{V_-} \bar{\mathbf{g}}_-. \quad (22)$$

We are using the notation

$$\tilde{\mathbf{g}} = \bar{\mathbf{g}} (g \bar{\mathbf{g}})^{-1} \quad (23)$$

that has the useful property

$$\mathbf{g} \tilde{\mathbf{g}} = \mathbf{1}_{N \times N}. \quad (24)$$

\bar{g} is the Hermitian conjugate of g . The proposed boundary interactions for the g_{\pm} bifundamentals are [23]

$$\mathcal{S}_{bdy}^{(gauge)} = \mathcal{S}_{kin}^{(1,1)} [g_+, V_+] + \mathcal{S}_{kin}^{(1,1)} [g_-, V_-] + \mathcal{S}_{CS}^{(1,1)} [\mathbf{V}_+^{g_+}, \mathbf{V}_-^{g_-}] - \mathcal{S}_{CS}^{(1,1)} [V_+, V_-] . \quad (25)$$

$\mathcal{S}_{kin}^{(1,1)}$ provides $\mathcal{N} = (1, 1)$ supersymmetric, gauge-invariant kinetic terms. In more explicit form

$$\begin{aligned} & \mathcal{S}_{kin}^{(1,1)} [g_+, V_+] + \mathcal{S}_{kin}^{(1,1)} [g_-, V_-] \\ &= -\frac{k}{2\pi} \int d^2x \int d\theta_{1+} d\theta_{2-} \left\{ \left(\bar{g}_+ \hat{\nabla}_-^{(+)} g_+ \right) \left(\bar{g}_+ \hat{\nabla}_+^{(+)} g_+ \right) + \left(g_- \hat{\nabla}_-^{(-)} \bar{g}_- \right) \left(g_- \hat{\nabla}_+^{(-)} \bar{g}_- \right) \right\} \end{aligned} \quad (26)$$

where the light-cone coordinates $x^{\pm} = x^0 \pm x^1$ were used, and $\hat{\nabla}_{\alpha}^{(\pm)}$ are the boundary versions of the chiral $\mathcal{N} = 2$ super-gauge-covariant derivatives

$$\hat{\nabla}_{\alpha}^{(\pm)} = e^{-V_{\pm}} D_{\alpha} e^{V_{\pm}} . \quad (27)$$

The last two terms on the rhs of equation (25) are a difference of two four-dimensional actions. This difference is a total derivative term that contributes Wess-Zumino-like interactions for g_{\pm} supported only on the two-dimensional boundary. To obtain this result one has to employ the property (24).

So far the total bulk-boundary action is

$$\mathcal{S}_{bulk}^{(1,1)} + \mathcal{S}_{bdy}^{(gauge)} = \mathcal{S}_{kin}^{(1,1)} [g_+, V_+] + \mathcal{S}_{kin}^{(1,1)} [g_-, V_-] + \mathcal{S}_{CS}^{(1,1)} [\mathbf{V}_+^{g_+}, \mathbf{V}_-^{g_-}] + \mathcal{S}_K^{(1,1)} + \mathcal{S}_W^{(1,1)} . \quad (28)$$

There are explicit couplings of the boundary degrees of freedom g_{\pm} with the vector multiplets V_{\pm} , but no couplings with the other bulk superfields, ϕ_{\pm} , A^a , B_a . From the string/M-theory discussion in section 2 we recall that the boundary is expected to break the bulk $SO(6) \times SO(2)$ R-symmetry to $SO(4) \times SO(2)$ and the action (28) does not have this property. This is already an indication that the open string theory of the D3-D5 intersection in configuration (3) involves additional boundary interactions.

From the open string theory of the configuration represented in Fig. 1 it is indeed clear that there is a cubic interaction on the two-dimensional boundary of the form

$$\mathcal{S}_{bdy}^{(matter)} = \int d^2x \int d\theta_{1+} d\theta_{2-} \text{Tr} \left[g_- \left(\hat{\phi}_- + \hat{\bar{\phi}}_- \right) \bar{g}_- + \bar{g}_+ \left(\hat{\phi}_+ + \hat{\bar{\phi}}_+ \right) g_+ \right] . \quad (29)$$

$\hat{\phi}_{\pm}$ denotes the $\mathcal{N} = (1, 1)$ projection of the bulk superfields ϕ_{\pm} on the two-dimensional boundary. For a succinct summary of boundary projections of superfields see appendix 3 of Ref. [14]. More precisely, in the particular context of eq. (29) by $\hat{\phi}_{\pm}$ we refer to the projection

$$\hat{\phi}_{\pm}(\theta_{1+}, \theta_{2-}) = \left(e^{V_{\pm}} \widetilde{\phi_{\pm}} e^{-V_{\pm}} \right) \quad (30)$$

where the notation $\hat{}$ refers to the notation of eq. (198) in [14]. The $\mathcal{N} = 2$ vector field exponents have been inserted to gauge-covariantize the derivatives normal to the boundary that appear in the $\hat{}$ projection. Notice that this cubic interaction has exactly the same form with a corresponding bulk-boundary interaction that appears in the field theory of the flat-space D3-D5 intersection [30, 31]. Although the physics of the flat-space D3-D5 intersection (without the additional 5-branes of the HW setup that we consider) is considerably different from the physics of our setup the uniqueness of this cubic interaction in [31] and its crucial role in the expected supersymmetry enhancement in that context, gives some confidence that (30) is the only extra interaction that we need to include at low energies.

As a further more direct check, we notice that the expected symmetries, e.g. invariance under the \mathbb{Z}_k transformations

$$A^a \rightarrow e^{2\pi i/k} A^a, \quad B_a \rightarrow e^{-2\pi i/k} B_a, \quad g_{\pm} \rightarrow e^{\mp 2\pi i/k} g_{\pm}, \quad (31)$$

do not allow cubic $U(M)$ -invariant interactions between g_{\pm} and the bi-fundamental fields A^a, B_a .

Moreover, the boundary interaction (29) implements the breaking of the R-symmetry

$$SO(6) \times SO(2) \rightarrow SO(4) \times SO(2) \quad (32)$$

that was anticipated from string/M-theory in section 2. This can be verified explicitly in the following way. In the three-dimensional bulk the UV action with the massive ϕ_{\pm} fields exhibits an $SU(2)_{diag}$ symmetry that rotates simultaneously the bottom components of the A^a and B_a superfields. In the IR the quartic superpotential (12) enhances this symmetry to $SU(2)_A \times SU(2)_B$ that rotates independently the fields A^a, B_a . These two $SU(2)$'s together with an independent $SU(2)_R$ symmetry that rotates the fields (A^1, B_1^*) combine to the $SO(6)$ of the bulk action. On the boundary the interaction (29) respects only the diagonal $SU(2)_{diag}$ symmetry of $SU(2)_A \times SU(2)_B$ and does not allow it to enhance in the IR. Hence, in the infrared we expect the theory to exhibit the overall global symmetry $SU(2)_{diag} \times SU(2)_R \times SO(2) \sim SO(4) \times SO(2)$.

Notice, that by integrating out the massive ϕ_{\pm} fields, setting $m = \frac{k}{8\pi}$, and using the identification (11), the boundary interaction (29) turns into the quartic interaction

$$\mathcal{S}_{bdy}^{(matter)} = \frac{4\pi}{k} \int d^2x \int d\theta_{1+} d\theta_{2-} \text{Tr}_{U(M)} \left[g_- \left(\hat{B}_a \hat{A}^a + \hat{\hat{A}}^a \hat{\hat{B}}_a \right) \bar{g}_- - \bar{g}_+ \left(\hat{\hat{A}}^a \hat{B}_a + \hat{\hat{B}}_a \hat{\hat{A}}^a \right) g_+ \right]. \quad (33)$$

Observe that further interactions of the components of the A^a, B_a superfields will be induced on the boundary by this integrating out procedure from the boundary terms included in $\mathcal{S}_K^{(1,1)} + \mathcal{S}_W^{(1,1)}$ according to the prescription (20).

3.3 Summary of the proposed bulk-boundary action

Collecting all the interactions in favor of which we argued above, we propose that the infrared limit of the bare bulk-boundary action

$$\begin{aligned}\mathcal{S}_{proposed}[g_{\pm}, V_{\pm}, \phi_{\pm}, A, B] &= \mathcal{S}_{bulk}^{(1,1)} + \mathcal{S}_{bdy}^{(gauge)} + \mathcal{S}_{bdy}^{(matter)} \\ &= \mathcal{S}_{kin}^{(1,1)}[g_+, V_+] + \mathcal{S}_{kin}^{(1,1)}[g_-, V_-] + \mathcal{S}_{CS}^{(1,1)}[\mathbf{V}_+^{g_+}, \mathbf{V}_-^{g_-}] + \mathcal{S}_K^{(1,1)} + \mathcal{S}_W^{(1,1)} + \mathcal{S}_{bdy}^{(matter)}\end{aligned}\quad (34)$$

describes the low-energy theory at the M2-M5 intersection (6). All the terms that appear in (34) were defined previously in the main text. We will not attempt to write out this action in components. In appendix A we present a more explicit form of the interactions in the case of a single M2-brane ending on an arbitrary number of M5-branes. The part that is hardest to expand in components is the non-abelian $\mathcal{N} = 2$ Chern-Simons action (14), which is written as a four-dimensional integral. There is such a non-abelian term, for general M , even in the abelian case of a single M2-brane, $N = 1$.

4 Outlook

In this paper we conjectured a specific action for the infrared theory in the M2-M5 intersection (6) with explicit $\mathcal{N} = 2$ supersymmetry in the bulk and explicit $\mathcal{N} = (1, 1)$ supersymmetry on the two-dimensional boundary. The boundary degrees of freedom and their interactions were deduced in large part from the open string theory of the type IIB Hanany-Witten configuration (3). Some evidence from the proposed interactions follows from the consistency of the constructions in [23]. We verified the expected global symmetries, and postulated that this action should exhibit the required $SO(4) \times SO(2)$ R-symmetry in the deep infrared. Accordingly we conjectured the enhancement of the boundary supersymmetry to $\mathcal{N} = (4, 2)$ for $k > 2$.

It would be very useful to find further checks of this preliminary proposal and eventually prove conclusively that it is the correct infrared description of M2-M5 physics. In this context, it would be interesting to explore the relation of this work with the Basu-Harvey equations [11]. It would also be interesting to explore relations with the work [22] upon compactification. In that respect, notice that the 2d boundary theories in [22] also include a pair of bi-fundamentals, which are analogous to our g_{\pm} .

Having a UV bare action is a first step towards the analysis of the quantum properties of the M2-M5 system. Generically this system is strongly coupled, but the introduction of the CS level k opens the possibility to go in weak coupling regimes. These are roughly regimes where the ratio N/k is small. It would be interesting to explore these regimes with perturbative techniques.

One of the issues that would be worth understanding better is whether the 2d boundary theory has a well-defined decoupling limit with a conserved 2d stress-energy tensor. One can then ask about the central charge of the boundary theory, and how it depends on the three parameters N, M, k . Our UV action introduces the massless boundary degrees of freedom g_{\pm} which belong in the bi-fundamental representation of $U(N) \times U(M)$. Hence, their number scales as NM in agreement with the anomaly considerations of Ref. [7]. In the IR the corresponding central charge can exhibit different scalings, similar to the reduction observed in the ABJM theory, where the N^2 UV scaling of the massless degrees of freedom reduces in the IR to the familiar $N^{3/2}$. It would be very interesting to see if the action that we propose has a consistent 't-Hooft like limit with $N, M \gg 1$ and the ratio $\lambda = M^2/N$ fixed, and if the central charge of the boundary theory scales in the large λ -limit as predicted by the blackfold supergravity analysis (2).

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A One M2-brane ending on M M5-branes

As a more explicit illustration of the proposed bulk-boundary interactions, in this appendix we consider in more detail the interactions that are packaged in the superspace action (34). We will focus on the relatively simpler case of a single M2-brane ending on an arbitrary number M of M5-branes. In this case the 3d bulk ABJM theory is abelian.

3d bulk action in $\mathcal{N} = 1$ superspace form

Our starting point is the bulk action (18)

$$\begin{aligned}
\mathcal{S}_{bulk} &= \mathcal{S}_{CS}[V_+, V_-] + \mathcal{S}_K[\phi_\pm, A, B, V_\pm] + \mathcal{S}_W[\phi_\pm, A, B] \\
&= \frac{k}{4\pi} \int d^3x d^4\vartheta \left[V_+ D^\alpha \bar{D}_\alpha V_+ - V_- D^\alpha \bar{D}_\alpha V_- \right. \\
&\quad \left. + \bar{\varphi}_+ \varphi_+ + \bar{\varphi}_- \varphi_- + \bar{A}_a e^{V_+} A^a e^{-V_-} + \bar{B}^a e^{V_-} B_a e^{-V_+} \right] \\
&\quad + \int d^3x d^2\vartheta \left[m(\varphi_+^2 - \varphi_-^2) + B_a A^a (\varphi_+ + \varphi_-) \right] + \text{c.c.}
\end{aligned} \tag{35}$$

We employ the $\mathcal{N} = 1$ superspace decomposition of the $\mathcal{N} = 2$ vector multiplets

$$V_\pm(\theta_1, \theta_2) = \Delta_\pm(\theta_1) + \theta_2 \Gamma_\pm(\theta_1) + \theta_2^2 (E_\pm(\theta_1) + D_1^2 \Delta_\pm) , \tag{36}$$

where Δ_\pm , E_\pm are $\mathcal{N} = 1$ real scalar superfields and Γ_\pm are $\mathcal{N} = 1$ spinor superfields. Following the conventions of [28] we use the notation $D_{1\alpha} = \partial_{1\alpha} + (\gamma^\mu \theta_1)_\alpha \partial_\mu$ for the $\mathcal{N} = 1$ superspace derivative with respect to the real Grassmann coordinates $\theta_{1\alpha}$ ($\alpha = \pm$ is a spinor index). For the $\mathcal{N} = 2$ chiral superfields we set

$$\phi_\pm(\theta_1, \theta_2) = \varphi_\pm(\theta_1) + i\theta_2 D_1 \varphi_\pm(\theta_1) + \theta_2^2 D_1^2 \varphi_\pm(\theta_1) , \tag{37}$$

$$A^a(\theta_1, \theta_2) = \mathbb{A}^a(\theta_1) + i\theta_2 D_1 \mathbb{A}^a(\theta_1) + \theta_2^2 D_1^2 \mathbb{A}^a(\theta_1) , \tag{38}$$

$$B_a(\theta_1, \theta_2) = \mathbb{B}_a(\theta_1) + i\theta_2 D_1 \mathbb{B}_a(\theta_1) + \theta_2^2 D_1^2 \mathbb{B}_a(\theta_1) . \tag{39}$$

Inserting these expansions in the $\mathcal{N} = 2$ expressions and performing the $\int d^2\theta_2$ integrals we obtain in $\mathcal{N} = 1$ form

$$\begin{aligned}
\mathcal{S}_{CS}[V_\pm] &= \frac{k}{4\pi} \int d^3x d^2\theta_1 \left[E_+ E_+ + \Gamma_+^\alpha W_{+\alpha} + \frac{1}{2} D_1^\alpha (D_{1\alpha} E_+ \Delta_+ - E_+ D_{1\alpha} \Delta_+) \right] \\
&\quad - \frac{k}{4\pi} \int d^3x d^2\theta_1 \left[E_- E_- + \Gamma_-^\alpha W_{-\alpha} + \frac{1}{2} D_1^\alpha (D_{1\alpha} E_- \Delta_- - E_- D_{1\alpha} \Delta_-) \right] .
\end{aligned} \tag{40}$$

We remind that the gauge-invariant field strength for a spinor multiplet Γ is

$$W_\alpha = \frac{1}{2} D^\beta D_\alpha \Gamma_\beta . \tag{41}$$

In passing we note that it would have been considerably harder to write out the corresponding expansion for the $\mathcal{N} = 2$ Chern-Simons action in general gauge in the non-abelian case. Also, note that in the so-called Ivanov gauge one sets $\Delta_{\pm} = 0$. This is not possible in the presence of the boundary unless we want to start with a partially broken super-gauge symmetry.

Similarly, for the kinetic terms S_K we obtain

$$\begin{aligned} \mathcal{S}_K[\phi_{\pm}, A, B, V_{\pm}] = \int d^3x d^2\theta_1 \left\{ \frac{1}{2} D_1^{\alpha} \left[\bar{\varphi}_+ D_{1\alpha} \varphi_+ + D_{1\alpha} \bar{\varphi}_+ \varphi_+ + \bar{\varphi}_- D_{1\alpha} \varphi_- + D_{1\alpha} \bar{\varphi}_- \varphi_- \right] \right. \\ + \frac{1}{2} e^{\Delta_+ - \Delta_-} D_1^{\alpha} \left[\bar{\mathbb{A}}_a D_{1\alpha} \mathbb{A}^a + D_{1\alpha} \bar{\mathbb{A}}_a \mathbb{A}^a \right] + \frac{1}{2} e^{\Delta_- - \Delta_+} D_1^{\alpha} \left[\bar{\mathbb{B}}^a D_{1\alpha} \mathbb{B}_a + D_{1\alpha} \bar{\mathbb{B}}^a \mathbb{B}_a \right] \\ + \left(E_+ + E_- + D_1^2 (\Delta_+ + \Delta_-) \right) \left(e^{\Delta_+ - \Delta_-} \bar{\mathbb{A}}_a \mathbb{A}^a + e^{\Delta_- - \Delta_+} \bar{\mathbb{B}}^a \mathbb{B}_a \right) \\ \left. - 2 \left[D_1^{\alpha} \bar{\varphi}_+ D_{1\alpha} \varphi_+ + D_1^{\alpha} \bar{\varphi}_- D_{1\alpha} \varphi_- + e^{\Delta_+ - \Delta_-} \nabla_1^{\alpha} \bar{\mathbb{A}}_a \nabla_{1\alpha} \mathbb{A}^a + e^{\Delta_- - \Delta_+} \nabla_1^{\alpha} \bar{\mathbb{B}}^a \nabla_{1\alpha} \mathbb{B}_a \right] \right\}. \quad (42) \end{aligned}$$

We used the $\mathcal{N} = 1$ super-gauge covariant derivative

$$\nabla_{1\alpha} = D_{1\alpha} - \frac{i}{2} (\Gamma_{+\alpha} + \Gamma_{-\alpha}). \quad (43)$$

Finally,

$$\begin{aligned} \mathcal{S}_W[\phi_{\pm}, A, B] = \frac{1}{2} \int d^3x d^2\theta_1 \left\{ m (\varphi_+^2 - \varphi_-^2) + 2m\theta_1 (D_1 \varphi_+ \varphi_+ - D_1 \varphi_- \varphi_-) \right. \\ + m \left(-D_1^{\alpha} (\varphi_+ D_{1\alpha} \varphi_+) + 2D_1 \varphi_+ D_1 \varphi_+ + D_1^{\alpha} (\varphi_- D_{1\alpha} \varphi_-) - 2D_1 \varphi_- D_1 \varphi_- \right) \\ + \mathbb{B}_a \mathbb{A}^a (\varphi_+ + \varphi_-) + 2\theta_1 D_1 \left((\varphi_+ + \varphi_-) \mathbb{B}_a \mathbb{A}^a \right) \\ - \theta_1^2 \left[\mathbb{B}_a \mathbb{A}^a D_1^2 (\varphi_+ + \varphi_-) - D_1 \mathbb{B}_a D_1 (\varphi_+ + \varphi_-) \mathbb{A}^a - D_1 \mathbb{A}^a D_1 (\varphi_+ + \varphi_-) \mathbb{B}_a \right. \\ \left. + (\varphi_+ + \varphi_-) \left(\mathbb{B}_a D_1^2 \mathbb{A}^a - D_1 \mathbb{B}_a D_1 \mathbb{A}^a + D_1^2 \mathbb{B}_a \mathbb{A}^a \right) \right] \left. \right\}. \quad (44) \end{aligned}$$

We will refrain from a further evaluation of the θ_1 integrals and the full expansion of these interactions in components. Nevertheless, it is already apparent from these expressions that there are several total-derivative terms that are supported on the boundary.

Boundary interactions

We restore half of the supersymmetry by adding suitable boundary interactions to the bulk action according to the rule (20)

$$\mathcal{S}^{(1,1)} = \int d^3x \left\{ d^2\theta_1 d^2\theta_2 \mathcal{L} - d^2\theta_1 \partial_2 \mathcal{L} \Big|_{\theta_2=0} + d^2\theta_2 \partial_2 \mathcal{L} \Big|_{\theta_1=0} - \partial_2 \partial_2 \mathcal{L} \Big|_{\theta_1=\theta_2=0} \right\}. \quad (45)$$

In the total bulk-boundary action (34)

$$\begin{aligned}\mathcal{S}_{proposed}[g_{\pm}, V_{\pm}, \phi_{\pm}, A, B] &= \mathcal{S}_{bulk}^{(1,1)} + \mathcal{S}_{bdy}^{(gauge)} + \mathcal{S}_{bdy}^{(matter)} \\ &= \mathcal{S}_{kin}^{(1,1)}[g_+, V_+] + \mathcal{S}_{kin}^{(1,1)}[g_-, V_-] + \mathcal{S}_{CS}^{(1,1)}[\mathbf{V}_+^{g+}, \mathbf{V}_-^{g-}] + \mathcal{S}_K^{(1,1)} + \mathcal{S}_W^{(1,1)} + \mathcal{S}_{bdy}^{(matter)}\end{aligned}\quad (46)$$

the first two terms on the second line are kinetic terms on the boundary and $\mathcal{S}_{bdy}^{matter}$ is a potential term on the boundary (29). The third term, $\mathcal{S}_{CS}^{(1,1)}[\mathbf{V}_+^{g+}, \mathbf{V}_-^{g-}]$, is the $\mathcal{N} = 2$ CS action for the non-abelian gauge group $U(M)$ with the boundary completion (45). The presence of a non-abelian boundary interaction, even for the abelian M2-brane theory, is a characteristic difference between our proposal and previous approaches.

The remaining two terms, $\mathcal{S}_K^{(1,1)}$, $\mathcal{S}_W^{(1,1)}$, on the second line of (46) are simple to write down. We collect the relevant expressions here. Once again, in order to keep the expressions somewhat compact we express everything in terms of $\mathcal{N} = 1$ superfields leaving the full expansion in components, that follows straightforwardly, implicit. For the kinetic interactions

$$\begin{aligned}\mathcal{S}_K^{(1,1)} &= \int d^3x d^2\theta_1 \left[\mathcal{L}_K - \partial_2 \left(\bar{\varphi}_+ \varphi_+ + \bar{\varphi}_- \varphi_- + e^{\Delta_+ - \Delta_-} \bar{\mathbb{A}}_a \mathbb{A}^a + e^{\Delta_- - \Delta_+} \bar{\mathbb{B}}^a \mathbb{B}_a \right) \right] \\ &+ \int d^3x \partial_2 \left[\mathcal{L}_K - \partial_2 \left(\bar{\varphi}_+ \varphi_+ + \bar{\varphi}_- \varphi_- + e^{\Delta_+ - \Delta_-} \bar{\mathbb{A}}_a \mathbb{A}^a + e^{\Delta_- - \Delta_+} \bar{\mathbb{B}}^a \mathbb{B}_a \right) \right]_{\theta_1=0},\end{aligned}\quad (47)$$

where \mathcal{L}_K is the integrand in eq. (42). Finally, for the superpotential interactions

$$\begin{aligned}\mathcal{S}_W^{(1,1)} &= \int d^3x d^2\theta_1 \left[\mathcal{L}_W - \frac{1}{2} \theta_1^2 \partial_2 \left(m(\varphi_+^2 - \varphi_-^2) + \mathbb{B}_a \mathbb{A}^a (\varphi_+ + \varphi_-) \right) \right] \\ &+ \frac{1}{2} \int d^3x \partial_2 \left[m(\varphi_+^2 - \varphi_-^2) + m \left(-D_1^\alpha (\varphi_+ D_{1\alpha} \varphi_+) + 2D_1 \varphi_+ D_1 \varphi_+ \right. \right. \\ &\left. \left. + D_1^\alpha (\varphi_- D_{1\alpha} \varphi_-) - 2D_1 \varphi_- D_1 \varphi_- \right) + \mathbb{B}_a \mathbb{A}^a (\varphi_+ + \varphi_-) \right]_{\theta_1=0} + \text{c.c.},\end{aligned}\quad (48)$$

where \mathcal{L}_W is the integrand in eq. (44).

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